Implementing Geometric Algorithms for Real-World Applications With and Without EGC-Support

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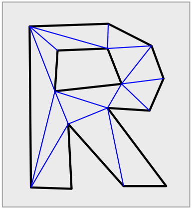
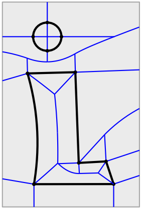
Outline

1. Three industrial codes and their design principles:

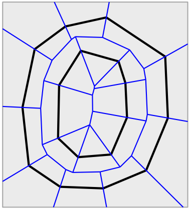
FIST



VRONI



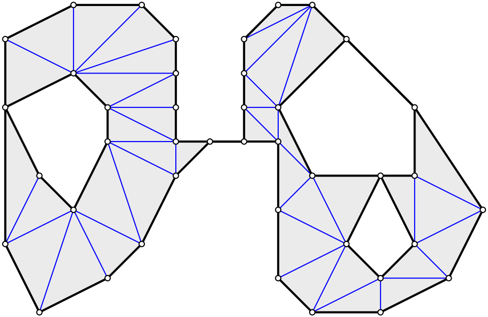
STALGO



2. Adding CORE and MPFR backend.
3. Open problems and future directions.

FIST

- Triangulates polygons with holes in 2D and 3D,
  - based on ear-clipping and
  - multi-level geometric hashing to speed up computation [Held, 2001a].
- Handles
  - degenerate input,



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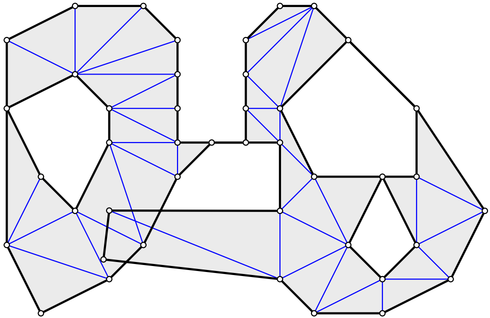

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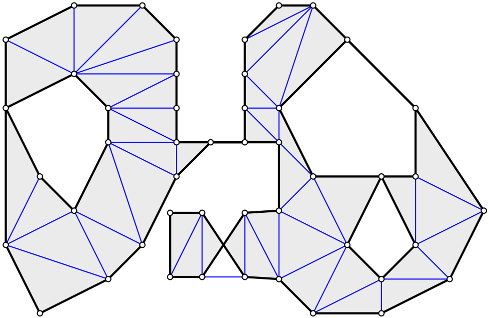

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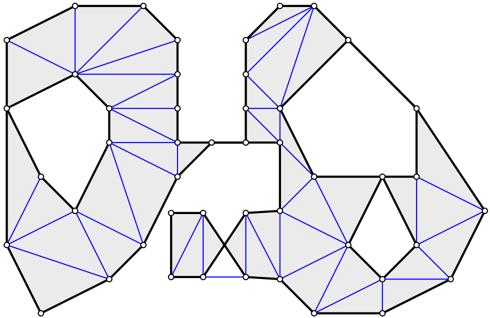
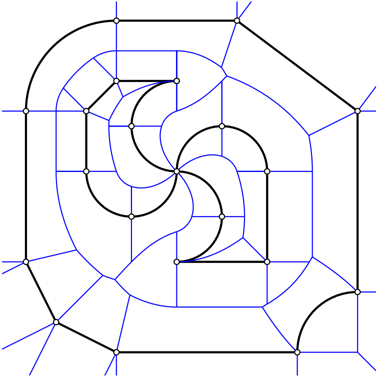


- No Delaunay triangulation, but heuristics to generate "decent" triangles.
- Typical applications in industry: triangulation of (very) large GIS datasets, triangulation of "planar" faces of 3D models.

Vroni/ArcVroni

- Computes Voronoi diagrams of
  - points, straight-line segments and circular arcs,
  - based on randomized incremental insertion and a topology-oriented approach [Held and Huber, 2009, Held, 2001b].



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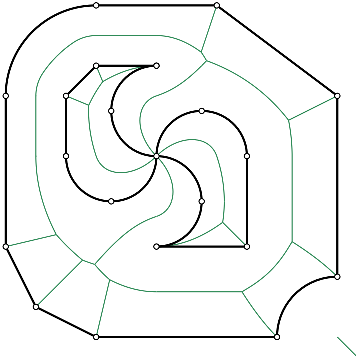

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  - (weighted) medial axis,

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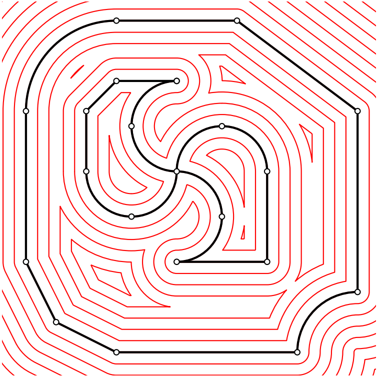

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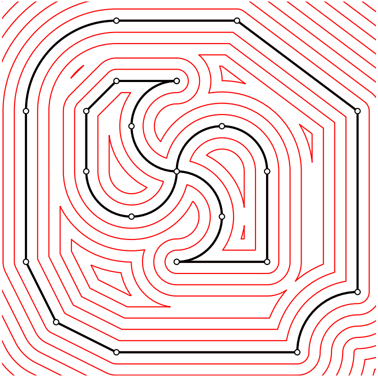
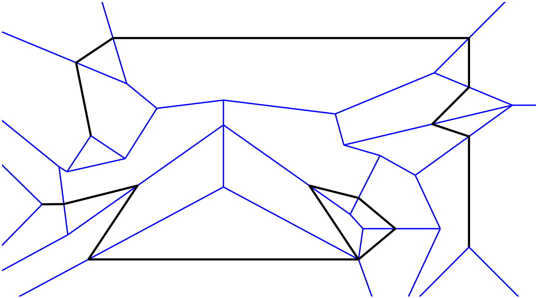


- Typical applications in industry: generation of tool paths (e.g., for machining or sintering), generation of buffers in GIS applications.

Stalgo

- Computing straight skeletons of
  - planar straight-line graphs,
  - based on a refined wavefront propagation using the motorcycle graph [Huber and Held, 2012, Huber, 2012].



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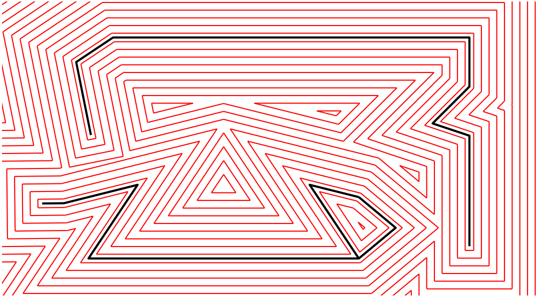

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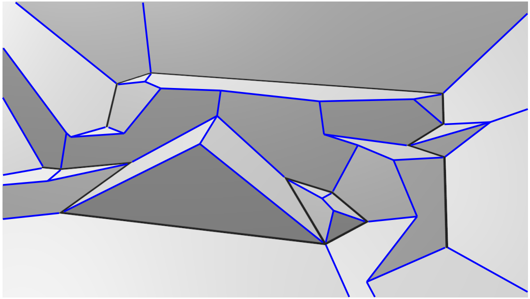
- roof models resp. terrains.



Success stories

- More than 100 commercial licenses world-wide for FIST, Vroni/ArcVroni and STALGO.
  - A few hundred Euros (for ArcVroni) up to a few thousand Euros/Dollars (FIST, VRONI, STALGO).

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  - A few hundred Euros (for ArcVroni) up to a few thousand Euros/Dollars (FIST, VRONI, STALGO).
- "Industrial-strength" implementations achieved:
  - Only a handful of bug reports in more than ten years
  - of heavy commercial and academic use, and lots of satisfied customers.

Datasets from industry

- Real-world data often means no quality at all:
  - brute-force simplifications / approximations of data,
  - data cleaned up manually and "visually",
  - etc.
- As a consequence:
  - All sorts of degeneracies, self-intersections, tiny gaps, etc.

General position must not be assumed.

Data sizes:

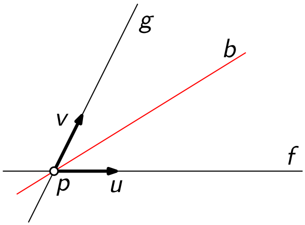
- From a few thousand segments/arcs in a machining application
- to a few million segments in a GIS application.

Efficiency requirements

- From real-time map generation on a smart phone
- to minutes of CPU time allowed on some high-end machine.
- In general, linear space complexity and a close-to-linear time complexity is expected.

Engineering principles: Use alternative computations

- Algebraically equivalent terms need not be equally reliable on fp arithmetic.
  - Check whether a computation becomes instable, and use an alternative approach.
- Sample application: Compute the bisector b between f and g.



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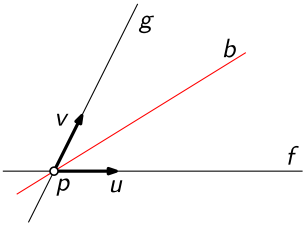

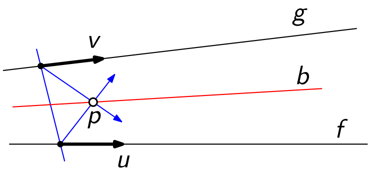

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- First used by Sugihara et alii [1992, 2000].
- Define topological criteria that the output has to meet.
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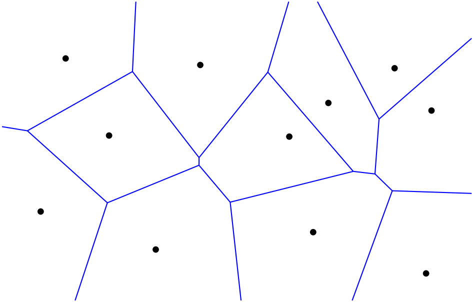

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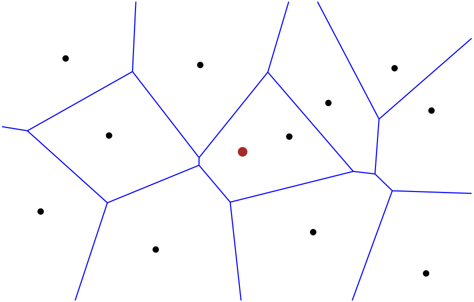

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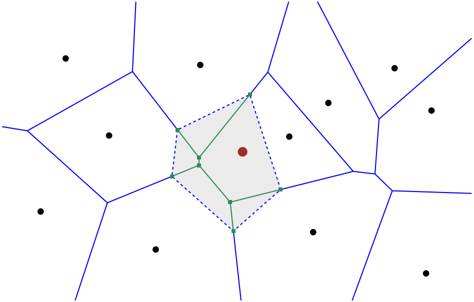

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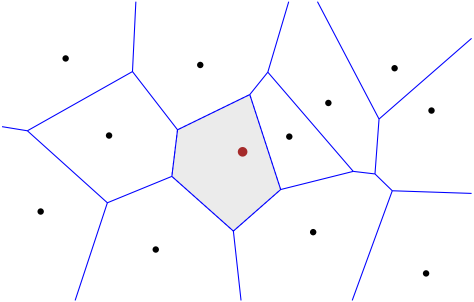
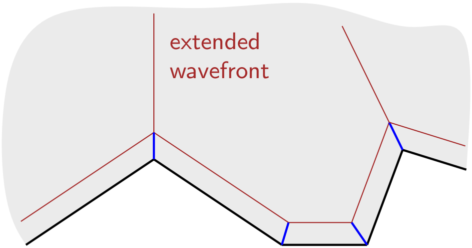


Engineering principles: Epsilon relaxation

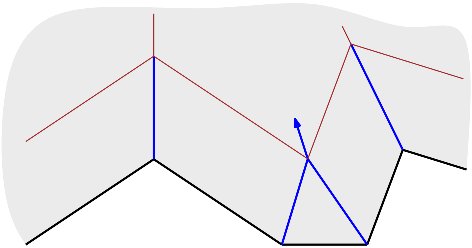
```
TypicalComputationalUnit()
begin
    ϵ ← ϵ_min                                   // Set ϵ to maximum precision
    while ϵ ≤ ϵ_max do
        result ← ComputeUnit(ϵ)                 // Compute some data
        if CheckResult(result, ϵ) then          // Topological/numerical checks
            return result
        else
            ComputeUnitReset()
            ϵ ← 10 · ϵ                          // Relaxation of epsilon
        end
    end
    if not CheckInputLocally() then             // Is input sound?
        CleanInputLocally()                     // Fix problems in the input
        RestartComputationGlobally()            // Restart from scratch
    else
        return ComputeUnitDesperateMode()       // Time to hope for the best
    end
end
```

Engineering principles: Avoiding geometric decisions



- Simulation of wavefront propagation, DCEL

Engineering principles: Avoiding geometric decisions



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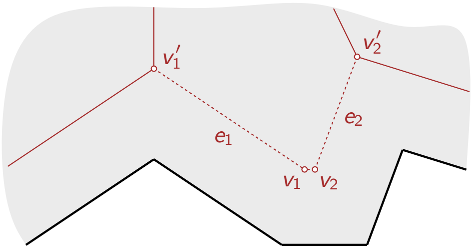

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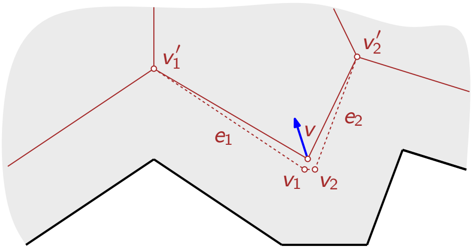

- Simulation of wavefront propagation, DCEL
- Straight-forward: remove e_1, e_2, v_1, v_2
  - Add v and relink it with v_1', v_2'.
  - Involves geometric decisions! And multiple events can occur simultaneously.

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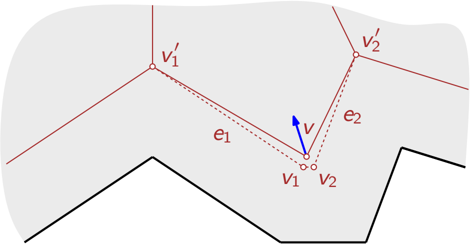

- Simulation of wavefront propagation, DCEL
- Straight-forward: remove e_1, e_2, v_1, v_2
  - Add v and relink it with v'_1, v'_2.
  - Involves geometric decisions! And multiple events can occur simultaneously.
- Better: remove v_1, v_2 but *repot* e_1, e_2 to v.
  - No geometric decisions involved.

Adding CORE backend

Canonical adaptions:

- Set `EPS` to `0`.
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More subtle problems encountered:

- `Expr::intValue()` rounds "inexact":
  - Rounds up or down, depending on expression tree.
  - Decision based on finitely many bits.
  - Work-around: migrate `intValue()` to `floor()`.

Adding CORE backend

Summary:

- FIST works with CORE.
- Vroni and Stalgo could not be executed.
  - Willi Mann's bug fixes and performance patches in CORE-2.1.
  - Still, several CPU-minutes did not suffice to determine sign of a single expression stemming from simple inputs.

Adding MPFR backend

Canonical adaptions:

- EPS needs to depend on precision.
  - We used a heuristic formula:

$$\texttt{EPS} := \epsilon_{\mathsf{fp}} \cdot 2^{-100\cdot(\sqrt{\mathsf{prec}/53}-1)},$$

where ϵ_{fp} is the former machine-precision EPS.

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Practical work required:

- MPFR is not shipped with a C++ wrapper.
  - Code that generates wrapper classes with the required operators overloaded.
- Partial C to C++ migration, as for CORE.

Experimental results: FIST

- 21175 polygons (w/ and w/o holes).
- Six arithmetic configurations:
  - fistFp, fistShew, fistCore, fistMp{53, 212, 1000}

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- Conclusion:
  - Shewchuck's predicates have negligible impact on speed.
  - fistMP* about 24× slower than fistFp.
  - fistCore about 60× slower than fistFp.

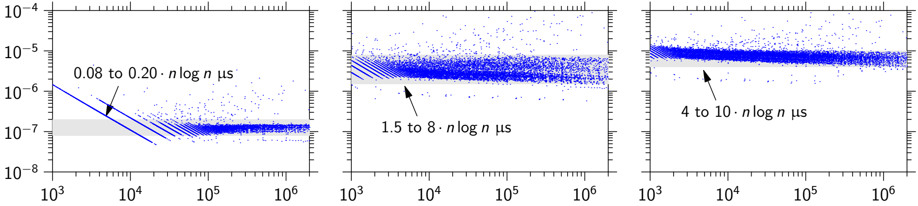

Figure: Runtime per seconds divided by $n \log n$. fistFp, fistMp212, fistCore.

Experimental results: FIST

Correctness of inexact configurations?

- Verification code:
  - Bentley-Ottmann, implemented with exact `mpq_t` from GMP.
- Take `0.1` as closest fp-number using `atof()`.
- No errors found!

Conclusion: Non-exactness no practical issue in pure fp applications.

Experimental results: Voronoi diagrams

- Vroni versus CGAL.
- 18787 polygons (< 100000 vertices)
- Six configurations:
  - vroniFp, vroniMp$\{53, 212, 1000\}$, cgvdFp
  - cgvdEx: CORE-based predicate kernel

Experimental results: Voronoi diagrams

- Conclusion:
  - vroniMp* about 50–70× slower than vroniFp.
  - cgvd* about 50–80× slower than vroniFp.
  - cgvdFp only 1.5× faster than cgvdEx.
    - Crashed on 937 datasets due to fp-exception.
  - On average, cgvdEx slightly faster than vroniMp*.
    - cgvdEx timings vary by a factor of 20.
    - A few cgvdEx results were numerically clearly wrong.

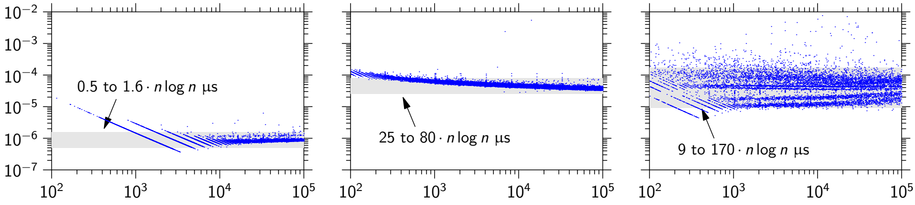

Figure: Runtime per seconds divided by $n \log n$. vroniFp, vroniMp212, cgvdEx.

Experimental results: Voronoi diagrams

Numerical precision of Voronoi nodes:

- **Deviation:** difference in the distances of a node to its defining sites.
- **Violation:** another site is closer to a node than defining sites.

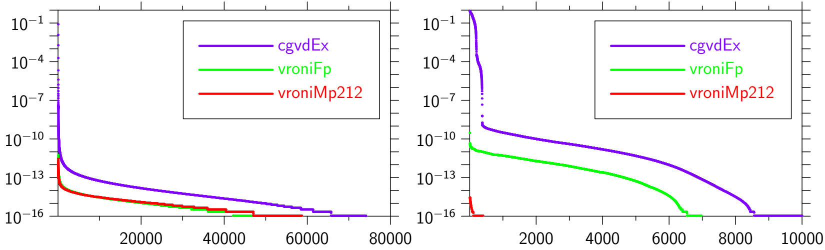

Figure: Left: Deviation. Right: violation

EGC: A simple case study

A function `test(N)`:

- Generate a shuffled array `A` with elements $\pm k_1, \ldots, \pm k_N$, with k_i being random integers.
- We build the sum `S` over `A`.
- How long does `S == Expr(0)` take?

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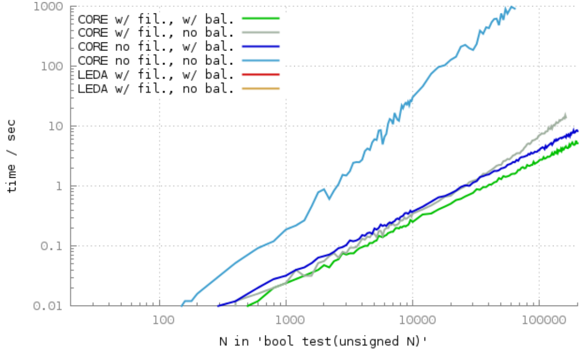
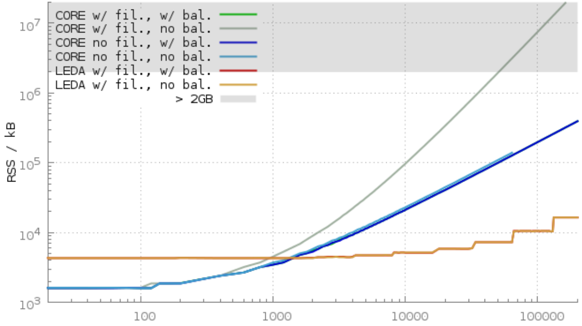
Results depend on the set-up:

- Are filters working?
- How is the sum built?
  - Naive for-loop, or
  - in a balanced fashion.

The “default case”: with filters, naive for-loop.



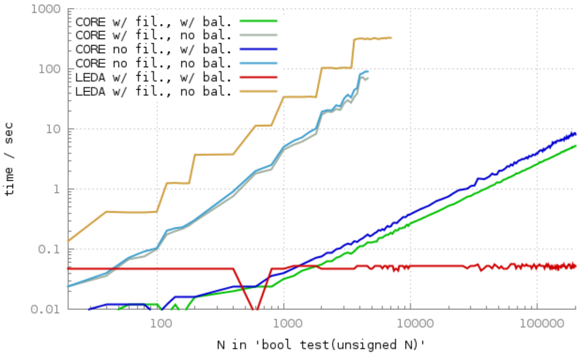


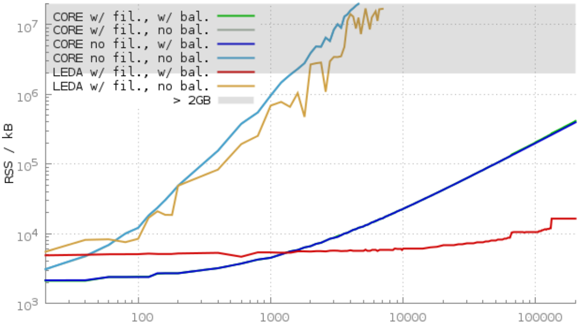
- CORE, naive sum:
  - $O(n^2)$ time
  - w/ filter: $O(n^2)$ mem
- LEDA: virtually zero runtime

EGC: a simple case study

What if we put stress on the filters?

- Add to the array `A` five times `sqrt(2)` and `-sqrt(2)`.
- How long will `S == Expr(0)` take now?





- naive sum:
  - $O(n^2)$ time
  - $O(n^2)$ mem
- balanced sum:
  - $O(1)$ or $O(n)$ time
  - $O(n)$ mem
  - filters have more impact

Disclaimer: Of course, these expressions will unlikely occur in real-world software.

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  - We might observe different complexities in terms of big-Oh.
  - On- and offline structural optimization strategies for expression trees are worth to be investigated.

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- Adding EGC support to non-trivial software a-posteriori can be extremely challenging.
  - Different programming styles due to focus on either numerical accuracy or awareness of expression trees.
  - EGC-aware programming right from the start is necessary.

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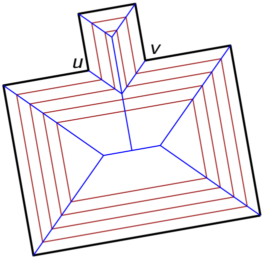
  - Different programming styles due to focus on either numerical accuracy or awareness of expression trees.
  - EGC-aware programming right from the start is necessary.
- Adding MPFR support is straight-forward
  - MPFR boosts numerical accuracy.
  - MPFR helps to distinguish numerical errors from logical bugs.
  - Precision-elevation instead of epsilon-relaxation?

Discontinuous problems and EGC

Straight skeletons can change discontinuously with the input:



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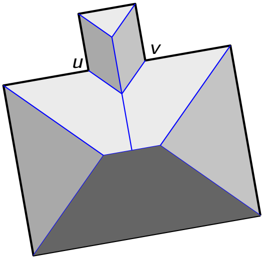

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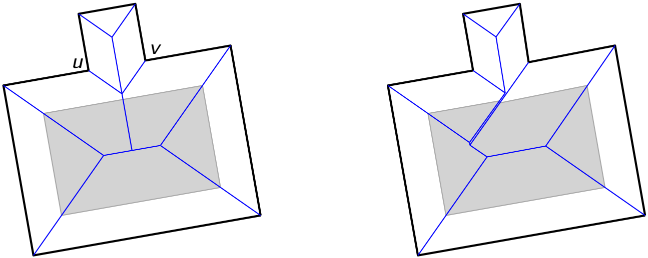

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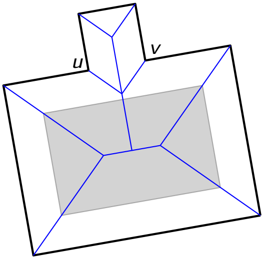

- The polygon is stored with finite precision to a file.
  - fp-codes are likely to produce the left skeleton/roof, which is intended.
  - EGC-codes produce the right skeleton/roof, which is undesired.

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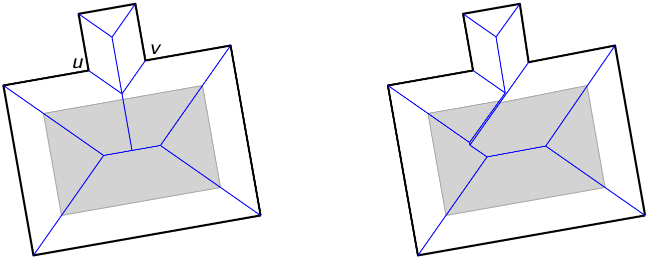

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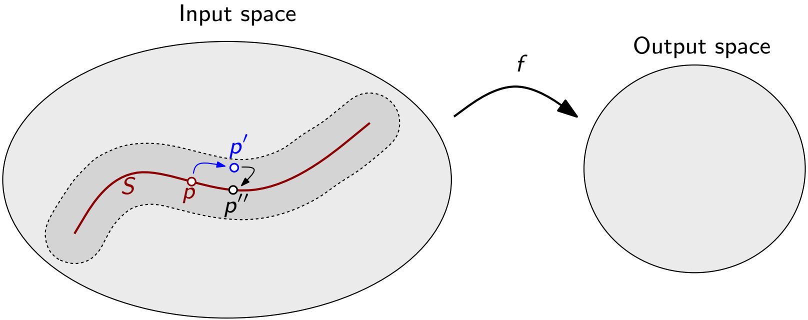
What is the lesser evil?

- Either waive EGC,
- Or forsake the desired output of the algorithm.

Discontinuous problems and EGC



- f is discontinuous on a sub-space S (red) of the input space.
  - "Reversed simulation of simplicity"?

A common yardstick

"Our algorithm runs in $O(n \log n)$ time in practice."

"Our implementation behaved reliable in our tests."

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However:

- Experiments often comprise only a few datasets.
- Datasets have no diversity.
- Different papers compare against different data, if at all.

A common yardstick

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  - How often did an implementation crash?
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- Enables a culture of extensive experimental evaluation.
  - Brings CG and industry closer together.
- Implementing reliable geometric codes requires testing.
  - An incentive to provide "gapless" and practial descriptions of algorithms.

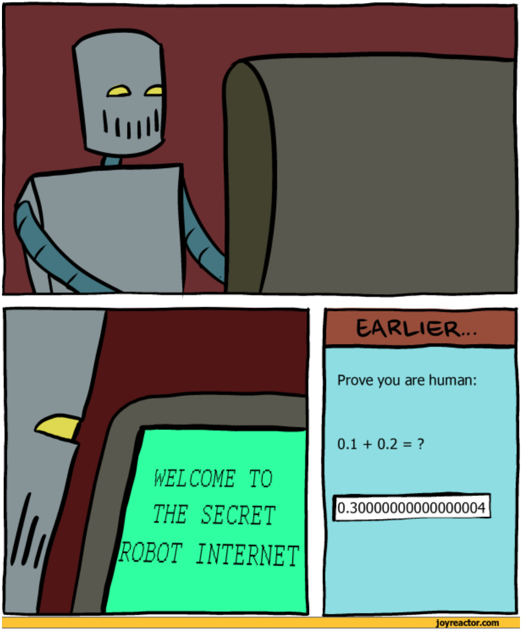

Figure: Taken from `http://joyreactor.com/post/818128`

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